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Review of Gauge Theories^{*}

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1. INTRODUCTION

Oftentimes, a new idea invites resistance. So it was with gauge theories of weak and electromagnetic interactions.¹ The idea of neutral current, whose existence had been speculated upon for nearly 15 years, and the existence of a fourth type of quark which was necessitated by phenomenology in a particular theoretical framework, were resisted by many. The discoveries of a series of new particles not only demolished this resistance, but liberated some theorists from the economy of having to do with as few a number of basic constituents of matter as possible. Roughly this is the situation with regard to model building in gauge theories of weak and electromagnetic interactions in recent months. Professor Harari described the situation very lucidly in his talk, and gave also a very nice introduction to the subject I shall address in this session. I would concur with him that there does not seem to be enough justification, based on phenomenology alone at this time, for postulating so many quarks and heavy leptons. On the other hand, some of the motivations for proliferating the number of quarks and leptons are more theoretical than phenomenological, and have to do with aesthetics of the theory, and with the desire to improve our understanding of nonleptonic weak interactions. Such speculations on more quarks and leptons are both healthy and desirable.

On the experimental side, we have learned immensely in the last year, not only from electron-positron colliding beams at SLAC, DESY and Frascati but also from neutrino physics at Fermilab reported here by Professor Rubbia: clearly, the discovery of dimuon events heralds the onset of new physics -- perhaps the same new physics Professors Gilman and Harari spoke of last week. Understanding this phenomenon is an important task both for experimenters and (gauge) theorists before the next conference.

In this talk, I shall first outline two outstanding issues in the theory of nonleptonic weak interactions. I shall then give a summary of analyses of higher order weak interactions based on gauge theory. An important aim of this discussion is to get an idea on the mass scale set by the fourth quark. This will be followed by a discussion on model-building which incorporates V + A currents. I shall emphasize motivations for such enterprises.

2. NONLEPTONIC WEAK INTERACTIONS - OUTSTANDING ISSUES

2.1. Observed $\Delta I = \frac{1}{2}$ and Octet Rules²

Experiment tells us that, typically, the size of a $\Delta I = 3/2$ amplitude is suppressed relative to that of a $\Delta I = 1/2$ amplitude:

$$\left| \frac{A(\Delta I = 1/2)}{A(\Delta I = 3/2)} \right| = 5\% .$$

However, the conventional (V-A) · (V-A) theory gives, for $|\Delta S| = 1$

transitions, a mixture of $\Delta I=1/2$ and $3/2$. Some mechanisms for the enhancement of the $\Delta I=1/2$ part (or octet) and/or the suppression of the $\Delta I=3/2$ part (or 27 part), are therefore necessary.

2.2 $K_S \rightarrow \pi^+ \pi^-$

In the conventional theory based on charged V-A currents, the octet part of the parity-violating nonleptonic weak interactions transforms like λ_6 :

$$\begin{aligned} [(V-A) \cdot (V-A)]_{\Delta S=\pm 1, p.v.} &= (\bar{s} \gamma_\mu \gamma_5 u)(\bar{u} \gamma^\mu d) \\ &+ (\bar{s} \gamma_\mu u)(\bar{u} \gamma^\mu \gamma_5 d) + h.c. \end{aligned}$$

On the other hand, $K_S \sim \bar{s} \gamma_5 d - \bar{d} \gamma_5 s$ transforms like λ_7 . As a consequence,³ $K_S \rightarrow \pi\pi$ is forbidden in the exact SU(3) limit, except through the (supposedly suppressed) 27 part of the Hamiltonian. The conventional wisdom has been that this decay proceeds through an SU(3) breaking effect and the 27 part of the Hamiltonian. This view, however, can be challenged.

2.3. What Happens in the Minimal Gauge Theory?

We define the minimal theory as the gauge theory of weak and electromagnetic interactions based on the minimal group $SU(2) \times U(1)$,^{4, 5, 6} which accommodates neutral current, and based on the minimal number of flavors, four, including charm.^{7, 8}

The situations with regard to the $\Delta I=1/2$ rule and the process

$K_S \rightarrow \pi\pi$ are the same in the minimal theory as in the conventional one.

The reason is that, to lowest order, the $\Delta S = \pm 1$ transitions are mediated by the product of two charged currents, which are identical in the two theories, except for the possible contributions from the charmed quark. Our intuition is that the charmed quark contributions to nonleptonic decays of ordinary hadrons are negligible, because ordinary hadrons contain very little charm-anticharm contents.

2.4 Asymptotic Freedom and Short-Distance Enhancement

It was proposed sometime ago by K. Wilson⁹ that the enhancement of the octet piece arises from short-distance singularity of the product of two currents.

Two currents can interact with each other via, e.g., color gluon exchange. This situation can modify the behavior of the product of two currents; roughly¹⁰

$$\begin{aligned} & \left(j_\mu(x) j_\nu(0) \right)_{\text{interacting theory}} \\ & \stackrel{x \rightarrow 0}{\sim} \left(j_\mu(0) j_\nu(0) \right)_{\text{free quark model}} V(x) \end{aligned}$$

where $V(x)$ is a kind of potential between the two currents due to gluon exchange. The effective strength of weak interaction is obtained from

$$\int d^4x \Delta_F(x; m_W) V(x) \quad .$$

Thus depending on whether $V(x) \rightarrow \infty$ or 0 as $x \rightarrow 0$, one gets either enhancement or suppression.

The distance relevant to weak interactions is typically of order $1/M_W < (38 \text{ GeV})^{-1}$. In an asymptotically free field theory (such as the SU(3) gauge theory of color gluons),¹¹⁻¹³ the behavior of $V(x)$ as such short distances can be reliably estimated. It is of the form^{14, 15}

$$V(x) = \left[1 + \frac{g^2(\mu)}{4\pi} b \ln \mu |x| \right]^\gamma$$

where $g(\mu)$ is the running coupling constant, μ is the running mass scale introduced to define the subtraction convention of the operator product $(j_\mu(0)j_\nu(0))_{\text{free quark model}}$, and b is a numerical constant.

For an octet combination of $j_\mu(x)j_\nu(0)$, the exponent γ turns out to be positive, 0.48 in the SU(3) gauge theory of color gluons, whereas for another octet and 27 combinations, γ is negative, -0.28. Thus, the effective weak Hamiltonian has the form

$$H_W = C_1 \mathcal{O}_8 + C_2 (\mathcal{O}_{27} + \mathcal{O}_{\bar{8}})$$

where, with reasonable choices of μ , $g(\mu)$ and M_W , one gets an enhancement (relative to a naive quark model) by a factor $\sim (7)^{0.48}$ for C_1 , and a suppression $\sim (7)^{-0.28}$ for C_2 .

It is my opinion that the short-distance enhancement in the color gluon gauge theory in the minimal model of weak interactions is not, by itself, sufficient to account for the observed ratio of the $\Delta I = 1/2$ to $3/2$ amplitudes. However, there will be additional enhancements of the matrix elements of the operator \mathcal{O}_8 relative to those of the operator \mathcal{O}_{27} , as

various arguments based on the quark model, duality and PCAC suggest. What is needed, I think, is a careful calculation of matrix elements of the product of two currents, where low frequency (or long distance) contributions are estimated by inserting the known hadron spectrum between the two currents and high frequency contributions by the renormalization group argument.

The short-distance enhancement is much bigger for the product of V-A and V+A currents,^{16, 17} and if there are more quarks in the color gluon theory.¹⁷

2.5 Charmed Particle Decays

In the minimal theory the decays of charmed particles are mediated by the piece of the weak Hamiltonian

$$\begin{aligned} &\sim \cos \theta_c [(\bar{s}c)(\bar{u}d) + (\bar{d}u)(\bar{c}s)] \\ &- \sin \theta_c [(\bar{d}c)(\bar{u}d) + (\bar{d}u)(\bar{c}d)] . \end{aligned}$$

To see which piece of this interaction is enhanced,¹⁸⁻²⁰ we examine the structure of the current \times current interaction in SU(4) which should be a good symmetry at short distances. The currents belong to a $\underline{15}$, so

$$\begin{aligned} [\underline{15}(\times)\underline{15}]_{\text{sym}} &= 1 \oplus 20 \oplus 84 \oplus 15_S \\ \text{SU(3) decomposition} &\left\{ \begin{array}{l} \Delta C = 0 \quad 8 \quad 8, 27 \\ |\Delta C| = 1 \quad 6, \bar{6} \quad 15, \bar{15}, \dots \end{array} \right. \end{aligned}$$

The $\underline{15}_S$ is actually absent in the product here because of the particular structure of charged currents in the minimal theory. The representation that is short-distance enhanced in $\underline{20}$, which contains only an octet. Thus, the short-distance enhanced charmed particle decays proceed via $\underline{6}$ and $\bar{\underline{6}}$.

A number of predictions on branching ratios and selection rules follow from the $\underline{6}, \bar{\underline{6}}$ character of charm decays.¹⁸⁻²⁰ Perhaps the most interesting²¹ is $D^+ \rightarrow \bar{K}^0 \pi^+$, which can be derived by the V spin zero character of the Cabibbo-favored, short-distance enhanced $\underline{6} \oplus \bar{\underline{6}}$ piece of the Hamiltonian.

In the article of Gaillard, Lee and Rosner²² on charmed particles it was assumed that the enhancement of the $\underline{6} \oplus \bar{\underline{6}}$ piece of the charm decay interaction is comparable to that of the $\underline{8}$ piece for strange particle decays.²³ Recently, this view was challenged by Ellis, Gaillard and Nanopoulos.²⁴ They argue that the short-distance enhancement is not as effective for charmed particle decays as for strange particle decays, due to the heavy charmed quark mass, and the arguments for low frequency enhancement of matrix elements are not applicable to the short-distance enhanced operator transforming like $\underline{6} + \bar{\underline{6}}$. Two consequences of this line of thought are that branching ratios and selection rules based on $\underline{6} + \bar{\underline{6}}$ dominance are not valid, and the inclusive branching ratio of a charmed particle decaying into $\mu + \text{anything}$ could be somewhat larger than estimated in GLR.²²

In fact, if there is no selective enhancement of a particular channel, we can estimate the branching ratio into $\mu + \text{anything}$ by a simple counting argument based on quark diagrams:

$$\frac{\Gamma(\text{charm} \rightarrow \mu + \text{anything})}{\Gamma(\text{charm} \rightarrow \text{all})} = \frac{1}{3 + 1 + 1} = 20\% .$$

The point I wish to convey here is that there is much room for speculation on details of charmed particle decays, and a leptonic branching ratio of $\sim 10\%$, which seems to be required for a "conventional" explanation²⁵ based on the minimal theory of the recently observed dimuon events, is quite reasonable and consistent with what we know today.

2.6 Implications on Hyperon Decays - Digression

Several authors^{26, 27} pointed out that in the exact SU(4) limit and in the minimal theory, there is another relation among the S-wave amplitudes for hyperon decays beyond the well-known SU(3) relation:^{28, 29}

$$2S(\Xi^- \rightarrow \Lambda \pi^-) = \sqrt{3} S(\Sigma^+ \rightarrow p \pi^0) + S(\Lambda \rightarrow p \pi^-)$$

$$4.08 \pm 0.04 = 4.04 \pm 0.05 ,$$

if we assume the $\underline{20}$ (in SU(4)) dominance. It is

$$S(\Lambda \rightarrow p \pi^-) = \frac{1}{\sqrt{3}} S(\Sigma^+ \rightarrow p \pi^0)$$

$$1.48 = 0.85$$

or

$$S(\Xi^- \rightarrow \Lambda \pi^-) = \frac{2}{\sqrt{3}} S(\Sigma^+ \rightarrow p \pi^0)$$

$$2.04 = 1.71.$$

The reason for this extra constraint is the antisymmetry of the $\Delta C = 0$ $|\Delta S| = 1$ weak interactions under exchange of the c- and u-quarks in the minimal theory. Since the c, u-symmetry (P-spin) is badly broken, it is not surprising that these latter relations are less well satisfied than the octet one.

3. HIGHER ORDER WEAK PROCESSES - MINIMAL THEORY

3.1 General Remarks

Typically without the GIM mechanism, the magnitude of second order processes is expected to be of order

$$G_F(G_F \Lambda^2) \sim G_F(G_F m_W^2) \sim G_F \alpha$$

in amplitude, where in the last approximate equality, we used the fact that in a unified theory of weak and electromagnetic interactions $G_F \sim \alpha/m_W^2$. This order of magnitude is much too big to explain the observed $K_L K_S$ mass difference, or the observed rate for $K_L \rightarrow \mu\bar{\mu}$.

The charm scheme introduces a new mass scale, m_c , the mass of the charmed quark. In fact with the GIM mechanism, the magnitudes of the processes $K^0 \leftrightarrow \bar{K}^0$, and $K_L \rightarrow \mu\bar{\mu}$, which would vanish if the c- and u-quarks were degenerate, are

$$G_F(G_F \Lambda^2) \sim G_F(G_F(m_c^2 - m_u^2)) \sim G_F \alpha \left(\frac{m_c}{m_W}\right)^2.$$

From the observed $K_L K_S$ mass difference, it is possible to find a bound on m_c .

3.2 $K_L K_S$ Mass Difference

In the free quark model, the $K^0 \leftrightarrow \bar{K}^0$ transition is described by Fig. 1. One obtains^{28,29}

$$\frac{m_L - m_S}{m_K} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \left(\frac{m_c}{m_W \sin \theta_W} \right)^2 \cos^2 \theta_c \sin^2 \theta_c \times \langle K^0 | \left[d \gamma_\mu \left(\frac{1-\gamma_5}{2} \right) s \right]^2 | K^0 \rangle.$$

The matrix element appearing on the right-hand side can be estimated by at least two ways [by inserting the vacuum between the two currents, or by relating it to $K^+ \rightarrow \pi^+ \pi^0$ by PCAC and SU(3)]. In any case, from the known $K_L K_S$ mass difference, one deduces

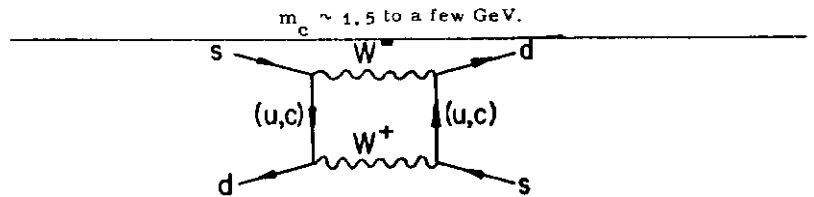


Fig. 1.

Diagram for $s + \bar{d} \rightarrow \bar{s} + d$ used to estimate the $K_L K_S$ mass difference.

In an asymptotically free gauge theory, it is possible to estimate³⁰⁻³² the effects of short distance gluon exchange using the renormalization group technique. In this case there are two scales relevant to the problem: referring to Fig. 2, we must have

$$|x-y| \approx |z-\omega| \lesssim \frac{1}{m_W},$$

$$\left| \frac{x+y}{2} - \frac{z+\omega}{2} \right| \lesssim \frac{1}{m_c}.$$

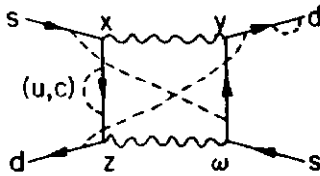


Fig. 2.

Operator products expansion of four currents relevant to the $K_L K_S$ mass difference calculation. Dotted lines are color gluons. The relevant region in configuration space is characterized by

$$|x-y| \approx |z-\omega| \leq \frac{1}{m_W},$$

$$\left| \frac{x+y}{2} - \frac{z+\omega}{2} \right| \leq \frac{1}{m_c}.$$

One might question whether one can evaluate the contribution from $|x+y)/2 - (z+\omega)/2| \sim 1/m_c$ assuming this distance is "short" on the scale of uncharmed hadrons, but it might be justified a posteriori by the observed scaling below charm threshold. In any case the net

result is that the effects of strong interactions make a negligible difference with respect to the free quark model calculation.

3.3 $K_L \rightarrow \mu \bar{\mu}$

For this process we examine the quark process $\bar{d} + s \rightarrow \mu + \bar{\mu}$.

There are two classes of diagrams depicted in Fig. 3. Each class goes as $G_F \propto (m_c/38 \text{ GeV})^2 \ln(m_W/m_c)$. A remarkable feature of this calculation is that the leading logarithmic terms in the two classes cancel.²⁸ Recently a number of authors³¹⁻³⁴ have repeated the calculation of Gaillard and Lee,²⁸ and found an error in the original calculation. The corrected expression is

$$iT(K_L \rightarrow \mu \bar{\mu}) = \frac{G_F^2 m_c^2}{4\pi^2} \cos \theta_c \sin \theta_c$$

$$\times \langle K_L | \bar{d} \gamma_\mu (1-\gamma_5) s + \bar{s} \gamma_\mu (1-\gamma_5) d | 0 \rangle \bar{\mu} \gamma^\mu \gamma_5 \mu.$$

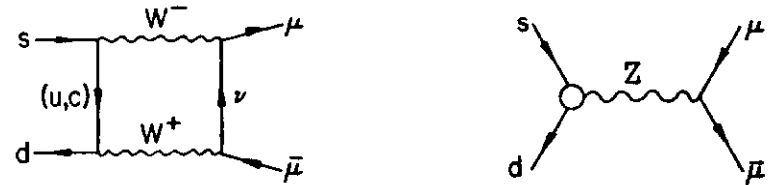


Fig. 3.

Two classes of diagrams for $s + \bar{d} \rightarrow \mu + \bar{\mu}$.

We believe³² that the leading logarithm cancellation persists even when the effects of strong interactions are taken into account. Thus, the

dominant mechanism for $K_L \rightarrow \mu\bar{\mu}$ is the conventional one:²⁸ $K_L \rightarrow$
(2 virtual γ 's) $\rightarrow \mu\bar{\mu}$.

4. MODEL BUILDING³⁵⁻³⁹

4.1 Motivations for V+A Currents and More Quarks

A. Product of V+A and V-A currents may produce terms of the form

$$\bar{s} \gamma_\mu (1 + \gamma_5) Q \bar{Q} \gamma^\mu (1 - \gamma_5) d + \text{h.c.},$$

or

$$\bar{s} \gamma_\mu (1 - \gamma_5) Q \bar{Q} \gamma^\mu (1 + \gamma_5) d + \text{h.c.},$$

where Q stands for a new, heavy quark.^{35, 36} The parity violating part of the above interactions transforms like λ_7 . There is then no longer the difficulty with the $K_S \rightarrow \pi\pi$ decay.

B. The above interactions are automatically octet under the ordinary flavor SU(3), since Q is a singlet. As mentioned before, the product of V-A and V+A currents suffers bigger short-distance enhancement.³⁶ Further it is possible to arrange the model so that the strangeness-changing (V-A) \times (V+A) interaction is not Cabibbo suppressed. If the (V-A) \times (V+A) term dominates over (V-A)², the octet rule follows.

However, there is a caveat. The above interactions may be rearranged to read

$$\bar{s} (1 \mp \gamma_5) d \bar{Q} (1 \pm \gamma_5) Q.$$

In a naive quark model, matrix elements of the above operators are expected to be small since ordinary hadrons contain very few Q and \bar{Q} quarks. An effective operator such as $\bar{s} \sigma_{\mu\nu} \lambda (1 \mp \gamma_5) d \cdot G^{\mu\nu}$, where $G^{\mu\nu}$ is the color gluon field strength, as might arise from the Q-quark loop, has a very small coefficient in an asymptotically free gauge theory.³⁶ Suffice it to say that whether the (V+A) \cdot (V-A) interaction of the usual strength produces nonleptonic weak interactions of the observed magnitudes remains controversial; I express my reservations.

C. There are also phenomenological considerations, e.g., anomaly in the γ -distribution in antineutrinos deep inelastic scattering; altering the charm particle decay scheme³⁵ (i.e., if charmed particle decays are dominantly mediated by e.g., $\bar{c} \gamma_\mu (1 + \gamma_5) d \bar{d} \gamma^\mu (1 - \gamma_5) u$, then there will be almost no strangeness carrying final states in these decays); dimuon events. As we have heard from Professor Wolfenstein today, the dimuon events of opposite charges may very well be explainable in terms of the minimal theory provided that charmed particles have $\sim 10\%$ branching ratio into muon channels (i.e., $\mu + \nu + \text{anything}$).

The dimuon events of the same sign are more problematic: whether they can be explained in terms of associated production of charmed pairs need to be examined, even though I would think so. An alternative explanation proposed³⁵ is a large amount of $\bar{D}^0(\bar{c}u) - D^0(c\bar{u})$ mixing. As in neutral K decays we write the 2×2 complex mass matrix in the $D_0 \bar{D}_0$ basis

$$M + i\Gamma.$$

There are two ways in which a large mixing might arise. The first is to allow a large number of common final states for D^0 and \bar{D}^0 decays. However, when there are many channels open, this will not, generally speaking, produce a large mixing, for

$$\Gamma_{11} = \Gamma_{22} = \sum_f |\langle D^0 | H_W | f \rangle|^2 = \sum_f |\langle \bar{D}^0 | H_W | f \rangle|^2,$$

whereas

$$\Gamma_{12} = \sum_f \langle D^0 | H_W | f \rangle \langle f | H_W | \bar{D}^0 \rangle,$$

and the off-diagonal Γ_{12} tends to be smaller than $\Gamma_{11} = \Gamma_{22}$ because the phases of various intermediate states in Γ_{12} tend to be distributed randomly. The second is to make off-diagonal elements of M large. Various dynamical schemes for enhancing M_{12} have been discussed by Kingsley, Treiman, Wilczek and Zee.³⁶

D. To some people, a vector-like theory of weak and electromagnetic interactions is esthetically pleasing.^{37, 38} A vector-like theory is a theory in which left-handed and right-handed chiral fermions appear symmetrically. Such a theory is parity conserving in the deep Euclidean region where fermion masses may be neglected. In such a theory, there is the attractive possibility of understanding the breaking of chiral symmetry (i.e., finite quark masses), parity-violation in weak interactions, the

Cabibbo angle and perhaps even the CP violation in a unified manner.⁴⁰

In a vector-like theory, the neutral current is parity-conserving.

E. Vector-like theories are anomaly-free. In fact it is in this context that vector-like theories were first discussed.⁴¹

4.2 Current Algebra Consideration

We have pointed out two possibilities of constructing a $(V-A) \cdot (V+A)$ interaction which transforms like an octet. They are

$$\bar{s} \gamma_\mu (1 \mp \gamma_5) Q \bar{Q} \gamma^\mu (1 \pm \gamma_5) d. \quad (4.2.1)$$

In addition there is the conventional $(V-A)^2$ term which is a mixture of $\underline{8}$ and $\underline{27}$. If the former dominates over the latter, one has an approximate octet rule.

Let Q^i and Q_5^i be isospin and axial isospin charges. Then

$$[Q^i \mp Q_5^i, H_W(\Delta I = \frac{1}{2})] \approx 0 \quad (4.2.2)$$

depending on whether the dominant $\Delta I = 1/2$ term (4.2.1) contains the right- or left-handed chiral d-quark, but

$$[Q^i + Q_5^i, H_W(\Delta I = \frac{3}{2})] \approx 0 \quad (4.2.3)$$

always, because the $\Delta I = 3/2$ part comes from the $(V-A)^2$ term.

Which sign in Eq. (4.2.2) does nature choose? It can be checked by comparing the amplitudes for $K \rightarrow 3\pi$ and $K \rightarrow 2\pi$ in a soft pion limit, using PCAC. The point is that

$$\begin{aligned} & \langle 2\pi, \pi^i | H_W(\Delta I = \frac{1}{2}) | K \rangle \\ & \xrightarrow{\text{soft } \pi^i} \frac{1}{f_\pi} \langle 2\pi | [Q_5^i, H_W(\Delta I = \frac{1}{2})] | K \rangle \\ & = \pm \frac{1}{f_\pi} \langle 2\pi | [Q^i, H_W(\Delta I = \frac{1}{2})] | K \rangle \end{aligned}$$

depending on which sign is chosen in Eq. (3.2.2), but we have always

$$\langle 2\pi, \pi^i | H_W(\Delta I = \frac{3}{2}) | K \rangle \xrightarrow{\text{soft } \pi^i} -\frac{1}{f_\pi} \langle 2\pi | [Q^i, H_W(\Delta I = \frac{3}{2})] | K \rangle,$$

and one knows the relative signs of the $\Delta I = 1/2$ and $3/2$ amplitudes in the $K \rightarrow 2\pi$ and $\rightarrow 3\pi$ decays.

Recently, Golowich and Holstein⁴² carried out a careful phenomenological analysis, and found that the dominant $\Delta I = 1/2$ part commutes with $Q^i + Q_5^i$, i.e., if it is of the form of (4.2.1), it contains the left-handed chiral d-quark and right-handed chiral s-quark.

4.3 Model for Quarks

In the minimal group scheme -- $SU(2) \times U(1)$, a vector-like theory which contains no singlet chiral fermions requires at least six quarks. If we insist that nonleptonic decays of hadrons are mediated by terms containing the left-handed chiral d-quark, then the model containing six quarks is more or less unique:^{36,37}

$$\begin{pmatrix} u \\ d_c \end{pmatrix}_L, \begin{pmatrix} c \\ s_c \end{pmatrix}_L, \begin{pmatrix} u' \\ d' \end{pmatrix}_L; \begin{pmatrix} u \\ d' \end{pmatrix}_R, \begin{pmatrix} u' \\ d \end{pmatrix}_R, \begin{pmatrix} c \\ s \end{pmatrix}_R.$$

Here, we ignore the possibilities of small admixtures such as

$c_L \rightarrow c \cos \alpha + u' \sin \alpha$, etc. This model is identical to Harari's scheme in (and only in) the quark contents.

A. The short-distance enhanced part of nonleptonic weak interactions comes from the term

$$\sin \theta_c \bar{s} \gamma^\mu (1 + \gamma_5) c \bar{c} \gamma_\mu (1 - \gamma_5) d. \quad (4.3.1)$$

(The enhancement factor for this term is³⁶

$$\left[1 + \frac{g^2(\mu)}{4\pi} \ln \frac{M_W}{\mu} \right]^{24/21}.$$

Even though Cabibbo suppressed, this term is perhaps more than enough to account for the required enhancement on short-distance behavior. The parity violating part of Eq. (4.3.1) transforms like λ_7 .

B. The interaction of (4.3.1) contains the left-handed chiral d-quark, and therefore, satisfied the current algebra constraint discussed in Sec. 4.2.

C. In this model, the dominant high frequency contribution to the $K_L - K_S$ mass difference arises from the diagram in Fig. 4.

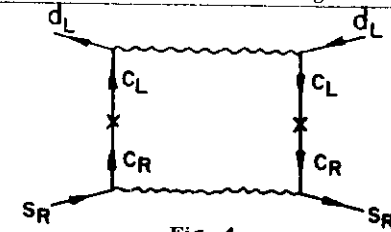


Fig. 4.

The process $s + \bar{d} \rightarrow \bar{s} + d$ in a vector-like six quark model; crosses denote charmed-quark mass insertions.

The diagram is of order

$$G_F \propto \left(\frac{m_c}{m_W \sin \theta_W} \right)^2 \cos \theta_c \sin \theta_c \ln \left(\frac{m_W}{m_c} \right) \\ \times \langle K^0 | [\bar{d}(1+\gamma_5)s]^2 | K^0 \rangle.$$

This has a large logarithm factor and there is no reliable way of estimating the matrix element. However, it is suppressed by the Cabibbo angle, and it is perhaps small enough with a reasonable c-quark mass, not to run afoul with the known $K_L K_S$ mass difference.

D. In this model, the $D^0 \bar{D}^0$ mixing is probably not too large, for the reasons discussed in Sec. 4.2. The high frequency contribution to M_{12} is probably of the order of magnitude of that of the $K_0 \bar{K}_0$ system.

4.4 Model for Leptons

There is no need for lepton-quark symmetry, for anomalies do not exist in a vector-like theory. On esthetic or other grounds, however, one might want it.³⁷ If there is to be no singlet under $SU(2) \times U(1)$, we must form six doublets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_L \\ L^- \end{pmatrix}_L; \begin{pmatrix} M_e \\ e \end{pmatrix}_R, \begin{pmatrix} M_\mu \\ \mu \end{pmatrix}_R, \begin{pmatrix} M_L \\ L^- \end{pmatrix}_R.$$

Of the three neutral right-handed chiral leptons, M_e , M_μ and M_L , at least two must be Majorana neutrals.

A. If M_e contains a Majorana neutral M_1 :

$$M_e = \cos \beta M_1 + \dots,$$

there is a disastrous consequence: lepton number is not conserved, and we would have nuclear double β -decays:

$$A \rightarrow B + e^+ + e^-.$$

In fact the upper bound on the rate of a nuclear double β -decay places a lower bound on the mass of M_e :

$$M_e \gtrsim \cos \beta \times 10^5 \text{ GeV},$$

(S. P. Rosen, et al⁴³).

B. If M_e and M_μ are some combinations of Majorana neutrals, then the following processes are allowed in second order: $K^\pm \rightarrow \pi^\mp + e^\pm + e^\pm$ ($\mu^\pm + e^\pm$, $\mu^\pm + \mu^\pm$).

C. If M_L is massive but lighter than L^- and ν_L massless, M_L decays radiatively. (See Fig. 5.) The decay rate can be read off the

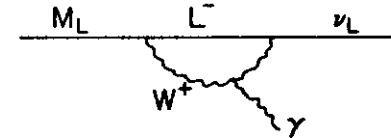


Fig. 5.

A diagram for the decay $M_L \rightarrow \nu_L + \gamma$.

calculations of Shrock⁴⁴ and Smith et al.,⁴⁵ on a similar process in the Georgi-Glashow model:

$$\Gamma(M_L \rightarrow \nu_L + \gamma) = \frac{1}{4\pi^2 (4\pi)^2} \left(\frac{G_F}{\sqrt{2}} \right)^2 \propto m(M_L) m(L^-)^2 m(M_L)$$

i.e.,

$$\tau(M_L) \approx 10^{-10} \text{ sec if } m(M_L), m(L^-) \approx 2 \text{ GeV.}$$

D. Suppose we arrange the masses of heavy leptons so that the multiplets are

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} L^0 \\ L^- \end{pmatrix}_L, \begin{pmatrix} M_e \\ e^- \end{pmatrix}_R, \begin{pmatrix} L^0 \sin \beta + M_\mu \cos \beta \\ \mu^- \end{pmatrix}_R, \begin{pmatrix} L^0 \cos \beta - M_\mu \sin \beta \\ L^- \end{pmatrix}_R$$

and $m(L^0) < m(L^-), m(M_2)$. Then the neutral heavy lepton L^0 is stable in lowest order. It can decay in second order,⁴⁶ through diagrams, one of which is shown in Fig. 6.

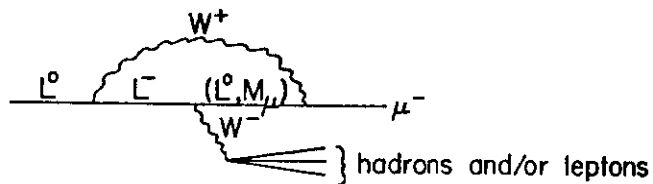


Fig. 6.
A diagram for L^0 decay.

However, as Professor Perkins discussed, evidence for the existence of a very long-lived neutral heavy lepton is not very strong.

5. CONCLUSION

It is my opinion that there is no compelling phenomenological reason to abandon the minimal gauge theory of weak and electromagnetic interactions (with the addition of one or more heavy lepton and its neutrino, perhaps -- See Harari's talk), or enlarge it by the addition of more quarks and V+A currents.

On the other hand this kind of theoretical speculations is both useful and healthy:

"In order to know the truth, it is necessary to imagine a million falsehoods" -- Oscar Wilde.

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Contrary to tradition, I shall not try to present an overview of the field, but will concentrate on a few subjects of topical interest.

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